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Technical Report

AN APPLICATION OF SOMMERFELD'S COMPLEX ORDER WAVE FUNCTIONS TO THE PROBLEM OF RADIATION FROM A DIELECTRIC COATED CONE

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ABSTRACT

Using the orthogonality relations of Sommerfeld's complex order wave functions, the exact solution for the problem of electromagnetic radiation from a circularly symmetric slot on the conducting surface of a dielectric coated cone is obtained. The results are valid for the near zone region as well as for the far zone region and they are applicable for arbitrary angle cones. It is noted that the technique used to solve this problem may be applied to similar type of problems involving conical structure, such as the diffraction of waves by a dielectric coated spherically tipped cone.

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I. INTRODUCTION

The problems of scattering of waves by perfectly conducting conical obstacle or radiation from such a structure have been considered by many authors [1-5]. The exact mathematical solution to the problem of the diffraction of waves by a finite, perfectly conducting cone has recently been obtained by Northover [6]. However, the corresponding solution for the diffraction by or radiation from a dielectric coated semi-infinite conical structure has not been found. It is the purpose of this paper to present the exact solution of the radiation from this dielectric coated structure. It is shown that certain mathematical difficulties can be overcome by the use of Sommerfeld's complex order wave functions [7] and their orthogonality properties.

II. FORMULATION OF THE PROBLEM

To analyze this problem, the spherical coordinates (r,θ,ϕ) are used. The geometry of this conical structure is shown in Figure 1. The vertex of the cone is taken to coincide with the origin of the spherical polar coordinates. To eliminate the singulaity at the vertex, a small perfectly conducting spherical boss of radius a with its center at the origin is situated at the tip of the cone. The outer boundary of the dielectric coated cone is assumed to coincide with $\theta = \theta_1$; the inner boundary is assumed to coincide with $\theta = \theta_o$. The dielectric coating has a permittivity of ϵ_1 , a permeability μ , and a conductivity of zero. It is assumed that this radiating structure is embedded in a homogeneous perfect dielectric medium $(\varepsilon_o, \mu; \sigma_o = 0)$, and that the applied electric field intensity across the slot which is located on the perfectly conducting conical surface, $\Theta = \Theta_o$, is circularly symmetric about the axis of the cone and linearly polarized in the radial direction.

Due to the symmetrical characteristics of this problem, all components of the electromagnetic field are independent of the azimuthal angle ϕ. For a TM wave, the non-vanishing components are E_r, E_Θ, and H_ϕ. The wave equation in spherical coordinates takes the form

$$\frac{\partial^2}{\partial r^2}(rH_\phi) + \frac{1}{r}\frac{\partial}{\partial\Theta}\left[\frac{1}{\sin\Theta}\frac{\partial}{\partial\Theta}(H_\phi \sin\Theta)\right] + k^2 r\, H_\phi = 0 \tag{1}$$

where $k^2 = \omega^2\mu\varepsilon$ and the steady state time dependence $e^{-i\omega t}$ has been assumed. Setting

$$H_\phi = i\omega\varepsilon\,\frac{\partial u}{\partial\Theta} \tag{2}$$

in equation (1) gives

$$(\nabla^2 + k^2)\,u(r,\Theta) = 0 \quad . \tag{3}$$

A possible solution of equation (3) is then $u(r,\Theta) = R(r)\,\Theta(\Theta)$ where R and Θ satisfy the differential equations

$$\frac{d}{dr}\left(r^2 \frac{dR}{dr}\right) + (k^2 r^2 - c)\, R = 0 \qquad (4)$$

$$\frac{d}{d\Theta}\left(\sin\Theta \frac{d\theta}{d\Theta}\right) + c \sin\Theta\, \theta = 0 \qquad (5)$$

in which c is the separation constant. If one chooses $c = \nu(\nu + 1)$ where ν may be a complex number, the solutions of equation (5) are the Legendre functions $\begin{Bmatrix} P_\nu(\cos\Theta) \\ Q_\nu(\cos\Theta) \end{Bmatrix}$. The corresponding solutions of equation (4) are the spherical Hankel functions $\begin{Bmatrix} h_\nu^{(1)}(kr) \\ h_\nu^{(2)}(kr) \end{Bmatrix}$.

The proper choice of these functions to represent the electromagnetic fields depends upon the boundary conditions. All field components must be finite in all regions (i.e., the region within the dielectric sheath and the region outside the sheath). In addition all field components for the radiated wave must satisfy Sommerfeld's radiation condition at infinity. Consequently the appropriate solution for the region inside the dielectric sheath is

$$u_s(r,\Theta) = \sum_\nu h_\nu^{(1)}(k_1 r)\,[A_\nu P_\nu(\cos\Theta) + B_\nu Q_\nu(\cos\Theta)] \qquad (6)$$

and that for the radiated wave

$$u_r(r,\Theta) = \sum_{\nu'} G_{\nu'}\, h_{\nu'}^{(1)}(k_o r)\, P_{\nu'}(\cos\Theta) \qquad (7)$$

where $k_1^2 = \omega^2\mu\epsilon_1$ and $k_o^2 = \omega^2\mu\epsilon_o$. A_ν B_ν and $G_{\nu'}$ are arbitrary constants to be determined by the boundary conditions. The summation is over all values of ν which are determined by the boundary condition on the spherical boss at the tip of the cone; i.e., the tangential electric field must vanish on the perfectly conducting spherical boss:

$$\frac{\partial}{\partial r}\left[r\, h_{\nu}^{(1)}(k_1 r)\right]\Big|_{r=a} = 0 \tag{8}$$

$$\frac{\partial}{\partial r}\left[r\, h_{\nu'}^{(1)}(k_o r)\right]\Big|_{r=a} = 0 \quad . \tag{9}$$

The roots of ν from equations (8) and (9) will be designated respectively by ν_n and ν'_m . It should be noted that $h_{\nu}^{(1)}(k_1 r)$ or $h_{\nu'}^{(1)}(k_o r)$ are orthogonal over the range $k_1 a$ to ∞ or $k_o a$ to ∞ respectively. (The proof is given in Appendix A.) It is because of this orthogonality property of these radial functions that they are so useful for the conical problems. The orthogonality characteristic of the Hankel functions with complex order was first investigated by Sommerfeld [7]. Hence these functions are also called Sommerfeld's complex order wave functions [8].

III. THE MATHEMATICAL SOLUTION

The boundary conditions require the continuity of the tangential electric and magnetic fields at the boundary surface, $\theta = \theta_1$. On the conducting surface, $\theta = \theta_o$, the tangential electric field must be zero everywhere except across the gap where it is equal to the applied field. Let the applied field be defined by

$$E_r^{app.} = E_o\, d(r)\, e^{-i\omega t} \tag{10}$$

where $d(r)$ is defined as follows:

$$d(r) = \begin{cases} 1 & \text{for } r_o < r < r_1 \\ 0 & \text{for } a < r < r_o \text{ and } r > r_1 \end{cases} \tag{11}$$

$|r_1 - r_o|$ is the gap width (see Figure 1). Expanding the applied field in terms of Sommerfeld's complex order wave functions gives

$$E_r^{app.} = \frac{1}{r} \sum_{\nu_n} L_{\nu_n} \nu_n(\nu_n+1) h_{\nu_n}^{(1)}(k_1 r) P_{\nu_n}(\cos \Theta_o) e^{-i\omega t} \tag{12}$$

where

$$L_{\nu_n} = \frac{1}{\nu_n(\nu_n+1) P_{\nu_n}(\cos \Theta_o) N_{\nu_n}(k_1 a)} \int_{r_o}^{r_1} E_o \, r \, h_{\nu_n}^{(1)}(k_1 r) \, d(k_1 r) \tag{13}$$

in which the normalizing factor is

$$N_{\nu_n}(k_1 a) = \int_{k_1 a}^{\infty} \left[h_{\nu_n}^{(1)}(k_1 r)\right]^2 d(k_1 r) \quad . \tag{14}$$

Expression (12) represents $E_r^{app.}$ for all values of r between a and ∞ .

Upon matching the tangential magnetic and electric fields at $\Theta = \Theta_1$, one obtains

$$i\omega\epsilon_o \sum_{\nu_m'} G_{\nu_m'} h_{\nu_m'}^{(1)}(k_o r) \left[\frac{d}{d\Theta_1} P_{\nu_m'}(\cos \Theta_1)\right]$$

$$= i\omega\epsilon_1 \sum_{\nu_n} h_{\nu_n}^{(1)}(k_1 r) \left[A_{\nu_n} \frac{d}{d\Theta_1} P_{\nu_n}(\cos \Theta_1) + B_{\nu_n} \frac{d}{d\Theta_1} Q_{\nu_n}(\cos \Theta_1)\right] \tag{15}$$

$$\sum_{\nu'_m} G_{\nu'_m} \nu'_m(\nu'_m + 1)\, h^{(1)}_{\nu'_m}(k_o r)\, P_{\nu'_m}(\cos \Theta_1)$$

$$= \sum_{\nu_n} \nu_n(\nu_n + 1)\, h^{(1)}_{\nu_n}(k_1 r) \left[A_{\nu_n} P_{\nu_n}(\cos \Theta_1) + B_{\nu_n} Q_{\nu_n}(\cos \Theta_1)\right] \quad (16)$$

It is noted that in contrast with the spherical and circular cylindrical boundary value problems, the boundary conditions can not be satisfied by equating each term of the series expansion. For the present case, the above equations must be satisfied for all values of r from $r = a$ to $r = \infty$. Consequently, the orthogonality properties of the radial function must be utilized to overcome the difficulty. Substituting the expansion

$$h^{(1)}_{\nu_n}(k_1 r) = \sum_{\nu'_m} \alpha_{\nu_n, \nu'_m}\, h^{(1)}_{\nu'_m}(k_o r) \quad (17)$$

into equations (15) and (16), and applying the orthogonality relations of the radial function, leads to the following expressions:

$$\frac{\varepsilon_o}{\varepsilon_1} G_{\nu'_m}\, g'_{\nu'_m} = \sum_{\nu_n} (A_{\nu_n} a'_{\nu_n} + B_{\nu_n} b'_{\nu_n})\, \alpha_{\nu_n, \nu'_m} \quad (18)$$

$$G_{\nu'_m} \nu'_m(\nu'_m + 1)\, g_{\nu'_m} = \sum_{\nu_n} (A_{\nu_n} a_{\nu_n} + B_{\nu_n} b_{\nu_n})\, \nu_n(\nu_n + 1)\, \alpha_{\nu_n, \nu'_m} \quad (19)$$

$$(\nu'_m = \nu'_o,\ \nu'_1,\ \nu'_2,\ \cdots)$$

where the abbreviations

$$a_{\nu_n} = P_{\nu_n}(\cos \Theta_1) \qquad a'_{\nu_n} = \frac{d}{d\Theta_1} P_{\nu_n}(\cos \Theta_1)$$

$$b_{\nu_n} = Q_{\nu_n}(\cos \Theta_1) \qquad b'_{\nu_n} = \frac{d}{d\Theta_1} Q_{\nu_n}(\cos \Theta_1)$$

$$g_{\nu'_m} = P_{\nu'_m}(\cos \Theta_1) \qquad g'_{\nu'_m} = \frac{d}{d\Theta_1} P_{\nu'_m}(\cos \Theta_1) \tag{20}$$

have been used. α_{ν_n,ν'_m} is given in Appendix B. Expressing B_{ν_n} in terms of A_{ν_n} gives (in matrix notation)

$$B_{\nu_n} = R^{-1}_{\nu_n,\nu'_m} D_{\nu'_m} \tag{21}$$

where $R^{-1}_{\nu_n,\nu'_m}$ is the inverse of the matrix

$$\left[R_{\nu'_m,\nu_n}\right] = \left[\left(\frac{\varepsilon_1}{\varepsilon_o} b'_{\nu_n} g_{\nu'_m} \nu'_m(\nu'_m + 1) - b_{\nu_n} \nu_n(\nu_n + 1) g'_{\nu'_m}\right) \alpha_{\nu_n,\nu'_m}\right]$$

and $D_{\nu'_m}$ is a column matrix

$$\left[\sum_{\nu_e} A_{\nu_e}\left(a_{\nu_e} \nu_e(\nu_e + 1) g'_{\nu'_m} - \frac{\varepsilon_1}{\varepsilon_o} a'_{\nu_e} g_{\nu'_m} \nu'_m(\nu'_m + 1)\right)\alpha_{\nu_e,\nu'_m}\right] .$$

Equation (21) can also be written in the form

$$B_{\nu_n} = \sum_{\nu_e} h_{\nu_n,\nu_e} A_{\nu_e} \tag{22}$$

where h_{ν_n,ν_e} are obtained using equation (21).

At the surface of the conducting cone, $\Theta = \Theta_o$, E_r in the dielectric sheath and equation (12) must be identically equal for all values of r between a and ∞. Therefore,

$$A_{\nu_n} P_{\nu_n}(\cos\Theta_o) + B_{\nu_n} Q_{\nu_n}(\cos\Theta_o) = L_{\nu_n} P_{\nu_n}(\cos\Theta_o) \qquad (23)$$

where L_{ν_n} is given by equation (13). Making the identification

$$d_{\nu_n} = P_{\nu_n}(\cos\Theta_o)$$

$$f_{\nu_n} = Q_{\nu_n}(\cos\Theta_o) \qquad (24)$$

and substituting equation (22) into equation (23), one finds

$$A_{\nu_n} d_{\nu_n} + f_{\nu_n} \sum_{\nu_e} h_{\nu_n,\nu_e} A_{\nu_e} = L_{\nu_n} d_{\nu_n} \qquad (25)$$

Solving for A_{ν_n} gives

$$A_{\nu_n} = \left[Q^{-1}_{\nu_n,\nu_m}\right]\left[L_{\nu_m} d_{\nu_m}\right] \qquad (26)$$

where $Q^{-1}_{\nu_n,\nu_m}$ is the inverse of the matrix

$$\left[Q_{\nu_m,\nu_n}\right] = \left[(d_{\nu_n} \delta_{\nu_n,\nu_m} + h_{\nu_n,\nu_m} f_{\nu_n})\right] \qquad (27)$$

and δ_{ν_n,ν_m} is the Kronecker delta which is equal to zero when $\nu_n \neq \nu_m$ and is equal to unity when $\nu_n = \nu_m$. $\left[L_{\nu_m} d_{\nu_m}\right]$ is a column matrix. With the knowledge of A_{ν_n} and B_{ν_n}, the coefficient $G_{\nu'_m}$ can easily be computed using either equation (18) or equation (19).

The electromagnetic fields in the dielectric shell and in the free-space are now completely determined. At large distances from the radiating source the asymptotic expressions for $h_{\nu'}^{(1)}(k_o r)$ which is

$$(e^{ik_o r}/k_o r)\, e^{-i(\nu'+1)\pi/2}$$

leads to

$$H_\phi^r = (i\omega\epsilon_o\, e^{ik_o r}/k_o r) \sum_{\nu'} G_{\nu'} \left[\frac{\partial}{\partial\theta} P_{\nu'}(\cos\theta) \right] e^{-i(\nu'+1)\pi/2} . \quad (28)$$

IV. CONCLUSIONS

By the use of Sommerfeld's complex order wave function and its orthogonality properties, the exact solution for the electromagnetic field excited by a slot on a dielectric coated, spherical tipped cone is obtained. The results are valid for the near zone (i.e., near the conical structure) as well as for the far zone. The influence of the presence of a dielectric sheath or a cold plasma sheath upon the electromagnetic field radiated from a spherically tipped cone can be computed. However, it should be noted that the computation is by no means trivial since the required Sommerfeld's complex order wave functions have not been tabulated, only certain limiting values are known at present. The tabulation of these functions in any detail is quite a involved though worthy project and can best be handled by an electronic computer group.

It is remarked here that the technique used in obtaining the solution for this problem is applicable to many similar type of problems involving conical structure, such as the diffraction of waves by a dielectric coated spherically tipped cone.

APPENDIX A. ORTHOGONALITY CHARACTERISTICS OF $h_\nu^{(1)}(kr)$

To show that the radial functions $h_\nu^{(1)}(kr)$ are orthogonal over the range $kr = ka$ to $kr = \infty$ [7], one notes that for any ν, say, ν_n

$$(kr)\left(d^2/d(kr)^2\right)\left(kr\, h_{\nu_n}^{(1)}(kr)\right) + \left((kr)^2-\nu_n(\nu_n+1)\right) h_{\nu_n}^{(1)}(kr) = 0 \qquad \text{(A-1)}$$

and for any other value of ν, say, ν_m

$$(kr)\left(d^2/d(kr)^2\right)\left(kr\, h_{\nu_m}^{(1)}(kr)\right) + \left((kr)^2-\nu_m(\nu_m+1)\right) h_{\nu_m}^{(1)}(kr) = 0 \qquad \text{(A-2)}$$

Multiplying the first equation by $h_{\nu_m}^{(1)}(kr)$ and the second by $h_{\nu_n}^{(1)}(kr)$ and integrating the difference from $kr = ka$ to $kr = \infty$, one gets

$$\begin{aligned}\left[\nu_m(\nu_m+1) - \nu_n(\nu_n+1)\right] \int_{ka}^{\infty} h_{\nu_n}^{(1)}(kr)\, h_{\nu_m}^{(1)}(kr)\, d(kr) \\ = \int_{ka}^{\infty} \Big[kr\, h_{\nu_n}^{(1)}(kr) \frac{d^2}{d(kr)^2}\left(kr\, h_{\nu_m}^{(1)}(kr)\right) \\ - kr\, h_{\nu_m}^{(1)}(kr) \frac{d^2}{d(kr)^2}\left(kr\, h_{\nu_n}^{(1)}(kr)\right)\Big]\, d(kr)\, . \qquad \text{(A-3)}\end{aligned}$$

Integrating the above by parts gives

$$\left[\nu_m(\nu_m+1) - \nu_n(\nu_n+1)\right] \int_{ka}^{\infty} h_{\nu_n}^{(1)}(kr)\, h_{\nu_m}^{(1)}(kr)\, d(kr)$$

$$= kr\, h_{\nu_n}^{(1)}(kr) \frac{d}{d(kr)}\left(kr\, h_{\nu_m}^{(1)}(kr)\right)$$

$$- kr\, h_{\nu_m}^{(1)}(kr) \frac{d}{d(kr)}\left(kr\, h_{\nu_n}^{(1)}(kr)\right)\Bigg|_{ka}^{\infty} . \qquad \text{(A-4)}$$

The terms on the right hand side of the equal sign are zero by virtue of the boundary condition (8) or (9) and the asymptotic behavior of $h_{\nu_n}^{(1)}(kr)$. Hence,

$$\int_{ka}^{\infty} h_{\nu_n}^{(1)}(kr)\, h_{\nu_m}^{(1)}(kr)\, d(kr) = \delta_{\nu_n,\nu_m} N_{\nu_n}(ka) \qquad \text{(A-5)}$$

where δ_{ν_n,ν_m} is the Kronecker delta and $N_{\nu_n}(ka)$ is a normalization factor which can be obtained from equation (A-4) by an application of de l'Hospital's rule for the limit $\nu_n \to \nu_m$.

APPENDIX B. FORMULA FOR $\alpha_{\nu_n,\nu_m'}$

Multiplying both sides of (17) by $h_{\nu_m'}^{(1)}(k_o r)$, integrating with respect to $k_o r$ from $k_o a$ to ∞, and using the orthogonality relation for the radial function (A-5), one obtains

$$\alpha_{\nu_n,\nu_m'} = \frac{1}{M_{\nu_m'}} \int_{k_o a}^{\infty} h_{\nu_n}^{(1)}(k_1 r)\, h_{\nu_m'}^{(1)}(k_o r)\, d(k_o r) \qquad \text{(B-1)}$$

where

$$M_{\nu_m'} = \int_{k_o a}^{\infty} \left[h_{\nu_m'}(k_o r)\right]^2 d(k_o r) . \qquad \text{(B-2)}$$

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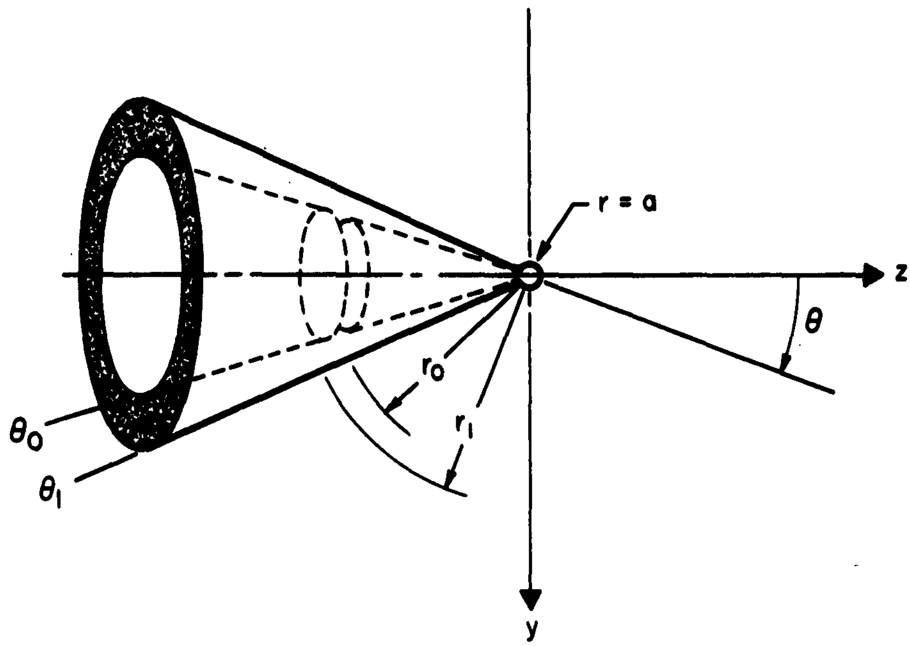

Figure 1 The Dielectric Coated Spherically Tipped Cone.

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